ciently low temperature dissociations lead to a "softening" of the Poisson adiabat as a result of the formation of atoms, and adiabats with S < 12 partially pass under the zero isotherm $\rho_m^0(p)$ for $p < p_*$. The beginning of dissociation (anomaly) on the S = 12 adiabat is lowered to $\sim$ 3 Mbar in agreement with the conclusions of the experimenters. We note that the construction of diagrams for equilibrium between purely molecular and purely atomic phases (each by the CCM) leads to an increase in p_* with increasing temperature, and the two-phase region on the S = 12 isentrope is almost the p = 5.4-Mbar isobar and almost the T = 0.41-eV isotherm.

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CALCULATIONS OF A HIGH-POWER UNDERWATER EXPLOSION TAKING VAPORIZATION INTO ACCOUNT USING THE GENERALIZED EQUATION OF STATE OF WATER

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The complexity of a theoretical investigation of high-power underwater explosions is mainly due to the fact that, according to estimates [1], at the initial instant gigantic pressures of up to 10^{10} bar and temperatures of tens of millions of degrees are reached. As a result, in the subsequent expansion, a large bubble is formed filled with water in the gaseous state and surrounded by concentric two-phase liquid–vapor layers. At the same time, in explosions using chemical explosives a cavity is formed containing mainly uncondensed products of the explosion, for which it is difficult to choose adequate equations of state suitable for calculating the action of the explosion [2-4]. In order to take into account in the calculations of a high-power underwater explosion the specific features connected with evaporation, dissociation of water, and ionization of its components, it is necessary to use the equation of state of water over a wide range of variation of the thermodynamic parameters. A fairly realistic model for calculating a high-power underwater explosion was set up in [5], where the equation of state of water describing the whole region enveloped by the shock wave was used, and the development of the explosion was considered, but the calculations were carried out for the initial stage of the formation of the two-phase region, and the effects of vaporization were not clarified in explicit form and were not analyzed.

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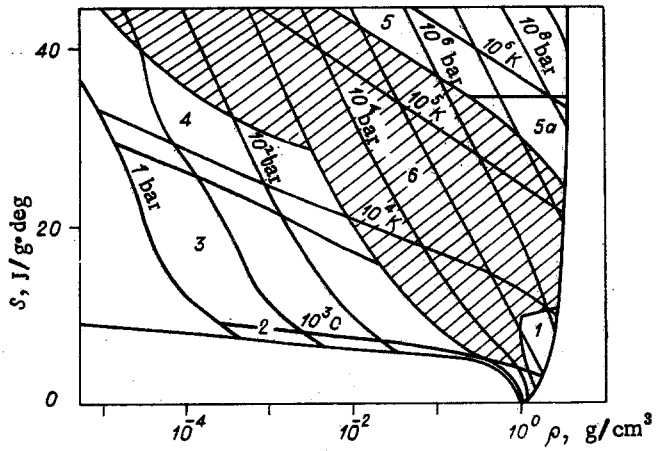

Fig. 1

In this paper we give a detailed presentation of the results [6] of the calculation by the finite-differences method of a high-power underwater explosion using the equation of state of water developed by one of the authors elsewhere [7]. The equation is given in analytical form and describes the whole range of variation of the thermodynamic parameters required to carry out such calculations. The "near" zone of the high-entropy states are considered in particular detail; these determine the volume of the vapor formed and also the plasma, i.e., the dimensions of the bubble at different instants of time.

1. Formulation of the Problem. The problem of the motion of the water as a result of the explosion can be solved in the one-dimensional approximation in the case of spherical symmetry. In Lagrange variables the equations of hydrodynamics for the motion of a medium can be represented in the form

$$u = \partial R/\partial t,\ \partial u/\partial t = -V\partial(p+q)/\partial R,$$
$$V/v = \partial R^3/\partial r^3,\ T\,\partial S/\partial t = -q\partial V/\partial t, \tag{1.1}$$

where t is the time; r, radial coordinate of the particle of liquid for t = 0; R(r, t), radial coordinate of this particle of liquid at the instant of time t; V, specific volume; v, its value for t = 0; u, velocity; p, pressure; S, entropy per unit mass; T, temperature; q, artificial viscosity, introduced, according to [8], in the form

$$q = \frac{c_0^2}{V}\left(\frac{\partial u}{\partial R}\right)^2(\Delta R)^2, \tag{1.2}$$

where c_0 is a dimensionless coefficient defining the width of the shock front, and ΔR is the spatial step for the difference scheme used, and for ∂u/∂R = 0 the value of q is assumed to be zero.

Equation (1.1) is supplemented by the equation of state

$$E = E(V, S) \tag{1.3}$$

(E is the specific internal energy). The relation between the pressure, the temperature, and the energy is specified by the well-known thermodynamic relations

$$p = -(\partial E/\partial V)_S,\ T = (\partial E/\partial S)_V. \tag{1.4}$$

Equations (1.1) and (1.3) together with relations (1.2) and (1.4) completely define the solution of the problem of the motion of the water in an underground explosion.

2. Equation of State of Water. The range of thermodynamic parameters required to calculate intense explosions is shown on the S–ρ diagram (where ρ is the density) (Fig. 1). Note that with the exception of the central zone of thermal heating the entropy for each element of volume increases only on the shock front and then remains constant. Hence, the

horizontal lines in Fig. 1, which are isentropic curves, determine the sequence of states through which the different elements of the volume pass as the pressure falls.

The equation of state obtained in [7] represents the analytically described thermodynamic potential which gives a complete and noncontradictory description of all the available information including data on the liquid–vapor phase boundary.

The equation of state of the form

$$E(V, S) = E_x(V) + \exp\,[w(\ln V, S)] \tag{2.1}$$

contains two characteristic functions, viz., the cold compression curve $E_x(V)$ and the thermal function w(lnV, S).

The partial derivatives of this thermal function, representing the slopes of the surface w along the coordinate axes lnV and S, are equal to the mean values of the Gruneisen coefficient and the inverse heat capacity [7]. Both these parameters are limited and vary smoothly, which ensures that the surface w(lnV, S) is smooth and that the interpolation is reliable.

The thermal function was approximated by the expression

$$w(\ln V, S) = w_0 + \left(\sum_{i,k} a_{ik}x^iS^k\right)\Big/\left(1 + \sum_{i,k} b_{ik}x^iS^k\right) + f(S), \tag{2.2}$$

where x = –lnV, and

$$f(S) = \left(\sum_i \varphi_i S^i\right)\Big/\left(1 + \sum_i \varkappa_i S^i\right). \tag{2.3}$$

The cold-compression curve $E_x(V)$ with density $\rho \geqslant \rho^*$(ρ^* = 0.9584 g/cm^3) was specified in the form

$$E_x(V) = E_c - a/V + \exp\left(\left(\sum_i \alpha_i x^i\right)\Big/\left(1 + \sum_i \beta_i x^i\right)\right). \tag{2.4}$$

For $\rho < \rho^*$ we use the expression

$$E_x(V) = E_c' - \exp\left(\left(\sum_i \alpha_i' x^i\right)\Big/\left(1 + \sum_i \beta_i' x^i\right)\right). \tag{2.5}$$

To find the approximation coefficients we used data from the following regions of the S–p diagram in Fig. 1:

1) data of dynamic measurements including recent investigations on the compressibility of ice of different densities [9]; 2) the region of numerous static investigations bounded by the isotherm T = 1000°C and the isobar p = 1000 bar; 3) the results of calculations of the dissociation of water molecules; 4) new calculations using the relations of the chemical equilibrium of the states of partial ionization and dissociation [7]; 5 and 5a) the regions of the parameters calculated in the Thomas–Fermi theory; and 6) the region where the equation of state is obtained by interpolation.

Numerical values of the coefficients occurring in (2.1)-(2.5) are given in [7].

The main difference from previous publications (e.g., [5]) is due to the use of new data on the dissociation and ionization of water, the dynamic compressibility of porous ice, comparative measurements of the parameters of the shock wave at a pressure of 14 Mbar, and highly accurate consistent calculations of the thermodynamic functions in the Thomas–Fermi theory.

The mean-square error of the approximation of all the data is 3-4%.

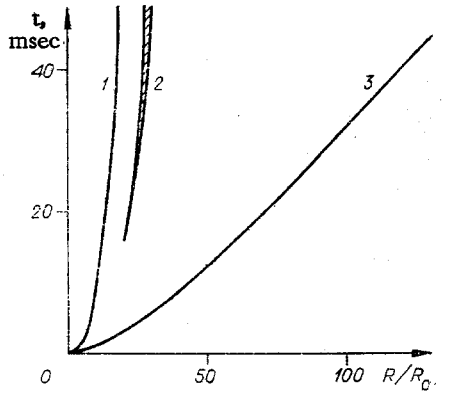

Fig. 2

The disagreement on the liquid–vapor phase boundary from the data given in [10] does not exceed several tenths of a percent, which enables the relations given in [10] to be used directly to describe the state of the vapor–liquid mixture.

In the gasdynamic calculation for the current values of S and V we check the condition $S(V) \geqslant S^*(V)$, where $S^*(V)$ is the entropy on the phase boundary [10]. If it was satisfied we used the equation of state (2.1)-(2.5), and if it was not the thermodynamic characteristics of the vapor–liquid mixture was calculated using the relations given in [10].

3. Initial Conditions and Method of Calculation. The calculations were carried out for a kiloton of underwater explosive with a counter pressure of 60 bar corresponding to a depth of $\approx$ 600 m. The initial conditions were specified by the "bursting" cavity model. This model assumes that at the initial instant the energy of the explosion (1 kton = $4.18 \cdot 10^{12}$ J) is uniformly distributed over an initial cavity of radius R_0 = 0.6 m (the initial density of the material in this cavity and in the medium surrounding it is taken to be 1 g/cm^3). The remaining thermodynamic parameters for $R \leqslant R_0$, obtained from the equation of state, are p = 23.5 Mbar, $T = 8.23 \cdot 10^5$°K, S = 3.50·10 J/g·deg, and $E = 4.03 \cdot 10^6$ J/g.

Equations (1.1) and (1.2) were approximated by a finite-difference scheme similar to that in [11].

The time step was chosen using the Neuman–Richtmyer stability criterion [8], in which the velocity of sound is found numerically at each step using the appropriate difference approximation.

In the initial cavity we took a uniform splitting over space into N cells, and outside it the dimensions of the cells were increased in a geometrical progression with ratio r.

4. Continuous Calculation. We will give some results of the continuous calculation for N = 10 and r = 1.04. The calculations were completed up to the instant of time 49 msec, the shock wave having traveled a distance of 89 m in this time, while the pressure had fallen to 0.5 kbar on the front and to 50 kbar in the central region, a reduction by more than five orders of magnitude compared with the initial pressure of 23.5 Mbar.

Figure 2 shows an R–t diagram for the calculation considered. The figure shows the boundary 1 of the initial cavity, increasing considerably with time, the vapor-plasma zone following after it, the two-phase region 2, formed at t = 16 msec, which at the end of the calculation occupies about 1 m, and the trajectory 3 of the shock wave.

Figure 3 shows the change in the pressure in the shock wave with distance for our calculation 1, the extrapolation 3 to the known experimental region 4, and the theoretical curve 2 from [5]. It can be seen that there is good agreement with the experimental data and that both calculations give similar results. Note that in [5] another equation of state was used, specified in tabular form; the initial stage is calculated using the homothermal self-model solution, and the method of characteristics is used.

If Fig. 4 we show on a p–S diagram the dependence of the pressure and the entropy in the two-phase region and its radii at certain fixed instants of time. At each given instant of time after the formation of the two-phase region the pressures in the latter are practically the same.

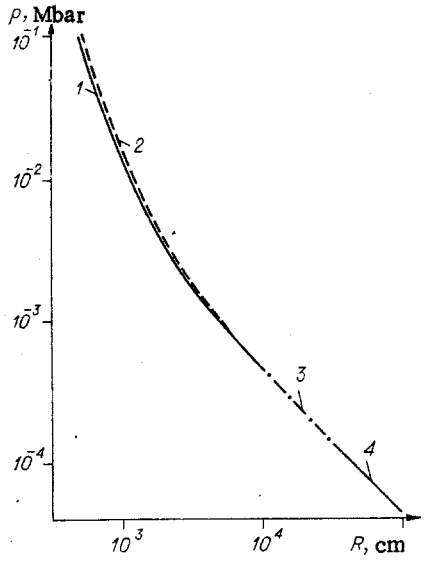

Fig. 3

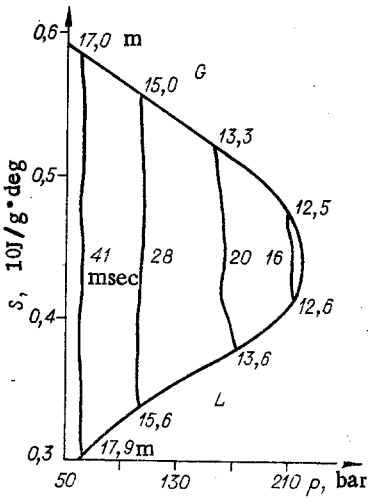

Fig. 4

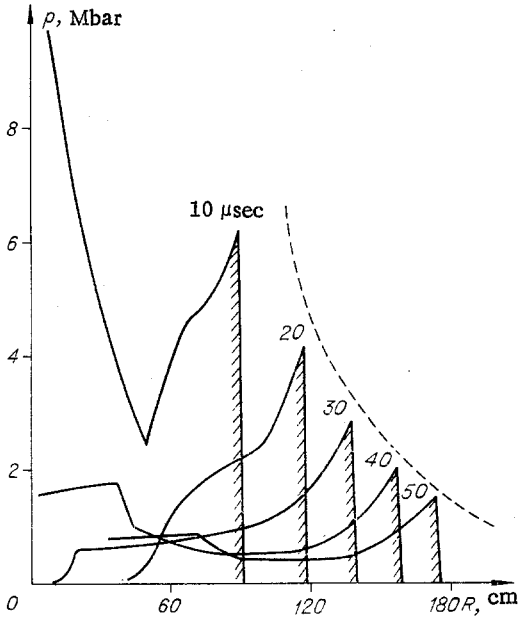

Fig. 5

5. Accurate Calculation of the Near High-Entropy Zone. For an unloading of up to 60 bar, the Hugoniot pressures, which determine the beginning and end of vapor formation, according to the equation of state used, are 420 kbar and 135 kbar, respectively, for entropies of 0.59·10 and 0.30·10 J/g·deg. According to our calculations, the lower values of these pressures and the entropies are achieved at a distance of 4.5 m. This radius bounds the zone of the high-entropy states, which predetermines the final dimensions of the vapor bubble. We carried out accurate calculations of the near zone up to a radius of 6 m.

The main version was carried out for N = 60 and r = 1.01. Figure 5 shows pressure profiles for several initial instants of time. We draw attention to the "wavy" form of the pressure variation in the central region. The material which forms the crest of the wave moves along its outer and inner slopes in different directions. The dashed curves show data from the calculations carried out in [5]. It can be seen that at differences greater than 1.6 m (at pressures less than 2 Mbar), the calculation of the initial stage using the homothermal solution [5] has much less effect and there is reasonably good agreement with our calculation.

TABLE 1

	Boundaries of the two-phase region, m		Volume of the two-phase region, $10^3 m^3$	
	G	L	G	L
Continuous calc.	17,0	17,9	20,5	23,8
Accurate calc.	16,3	17,2	18,1	21,1

Calculations for different spatial splittings, i.e., for different N and R, show good convergence of the calculation. At the same time, the difference between the main calculation of the near zone and the data of the continuous calculation are considerable.

To estimate the accuracy of the dimensions of the bubble obtained in the continuous calculation we carried out accurate calculations of the position of the boundary of the two-phase region. The entropy of each element of the medium, calculated fairly accurately at the initial stage, remains constant. The corresponding specific volumes were obtained in the single-phase region using the equation of state, and in the two-phase region using the parameters of the line of equilibrium. By integrating them we found the boundaries of the two-phase region, when the pressure in the central zone is constant and equal to 60 bar. The results of the comparison of both calculations are shown in Table 1.

As can be seen from Table 1, the radii of the boundaries are higher in the direct calculation by 4%, while the volumes are higher by 12%. The accuracy of the direct calculations can be increased considerably by using finer spatial splitting.

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